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A ROUGH EARTH SCATTERING MODEL FOR MULTIPATH PREDICTION

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A ROUGH EARTH SCATTERING MODEL FOR MULTIPATH PREDICTION

1. INTRODUCTION.

The most important phenomena to be considered in a model of radio wave communication between earth satellites are scattering from the surface of the earth. In Section 2 we describe in detail the model we have derived and implemented on a computer to predict the field received after reflection from a rough, spherical earth. In Section 2.1 our discussion follows that of Beckman and Spizzichino[1] but differs from theirs and from all others, Section 2.2, in two important respects. First, the scattering integrals are not approximated, but are computed numerically; second, the domain of integration is the appropriate region on the surface of the earth. The work of Beckman and others deals with the flat earth approximation which is inaccurate for satellite to satellite communication.

The results of the calculations are discussed in Section 3. Calculations have been performed at VHF frequencies and for terrain which could be described as marshy land. Various different altitudes are considered as well as several values of the physical parameters which describe the roughness of the earth. The results show that:

a. Rough surface scattering calculations must be performed over a spherical earth when satellites are involved.

b. There is a definite dependence on the values of the roughness, σ, and the correlation length, T. The ratio of σ/T can not be taken as an arbitrary small number as has been done previously in some calculations.[6]

1. -- Continued.

c. For some types of pulsed communication systems, the multipath effect is largest for incident angles (the angle of the incoming ray measured from the normal to the surface) which are approximately 80 degrees. Some previous analysis have considered the worst case to be at an incident angle of zero degrees.

2. THE SCATTERING OF RADIO WAVES FROM THE SURFACE OF THE EARTH.

In this section we first summarize the work of Beckman and Spizzichino[1] which describes a method for obtaining the average value and the mean square value of the amplitude of an electric field received at a point after reflection from a rough surface. It is assumed that the wave illuminating the surface is a plane wave and that the surface is a plane except for random irregularities which are normally distributed. This theory is then applied to the problem of a spherical wave impinging upon the surface of the earth by dividing the illuminated portion of the surface into small elements such that:

a. Each element of surface δS is large compared with the horizontal dimensions of the irregularities

b. Each element of surface is sufficiently small so that the incident angle of the incoming wave does not vary appreciably from point to point on that element, i.e., the spherical wave may be approximated by a plane wave over each element.

The contributions from each element of surface are added together to yield the total field reflected from the surface of the earth.

Section 2.1 summarizes Chapters 3 and 5 of Beckman's book; the application to scattering from the surface of the earth is made in Section 2.2.

2.1 Scattering From Elemental Areas.

Let a surface S be given by

$$z = \zeta(x, y)$$

where x, y, and z are cartesian coordinates. We will consider ζ a random process of the space variables x and y with zero mean, and the statistics of the scattered field will be determined by taking certain averages over this random surface.

In order to obtain expressions for the average amplitude and the average mean square value of the field scattered up to the receiver, we require a knowledge of the first and second order statistics of the random surface, i.e., the distribution function

$$F_1\left(\zeta_1\right) = P\left(\zeta(x, y) \leq \zeta_1\right)$$

where P denotes the probability measure, and the second order joint distribution

$$F_2\left(\zeta_1, \zeta_2\right) = P\left(\zeta\left(x_1, y_1\right) \leq \zeta_1, \zeta\left(x_2, y_2\right) \leq \zeta_2\right)$$

(The dependence of F_2 on the points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ has been suppressed.) The probability density functions are defined by

$$p_1\left(\zeta_1\right) = \frac{\partial F_1\left(\zeta_1\right)}{\partial \zeta_1}$$

2.1 -- Continued.

and

$$P_2\left(\zeta_1, \zeta_2\right) = \frac{\partial^2 F_2\left(\zeta_1, \zeta_2\right)}{\partial\zeta_1 \, \partial\zeta_2}$$

We assume that the surface may be adequately described by the normal distribution (although other distributions have been used); we obtain therefore

$$P_1\left(\zeta_1\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \, e^{-\frac{\zeta_1^2}{2\sigma^2}} \tag{1}$$

and

$$P_2\left(\zeta_1, \zeta_2\right) = \frac{1}{2\pi\sigma^2\sqrt{1-C^2}} \, e^{\left[-\frac{\zeta_1^2 - 2C\,\zeta_1\zeta_2 + \zeta_2^2}{2\sigma^2\left(1-C^2\right)}\right]} \tag{2}$$

where σ^2 is the variance, and $C = C(\tau)$ is the correlation coefficient, τ being the distance between $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$.

2.1 -- Continued.

We choose

$$C(\tau) = e^{-\frac{\tau^2}{T^2}}$$

where T is called the correlation distance.

Let $\vec{E}_1$ be the field incident on the surface $\zeta(x, y)$, and let its amplitude be a plane wave:

$$E_1 = |\vec{E}_1| = e^{i \vec{k}_1 \cdot \vec{r}}$$

where the time factor $e^{-i\omega t}$ has been suppressed,

$$\vec{k}_1 = \frac{2\pi}{\lambda} \frac{\vec{k}_1}{k_1}$$

is the propagation vector, λ denotes wavelength, and

$$\vec{r} = (x, y, z)$$

is the radius vector, as indicated in Figure 2-1.

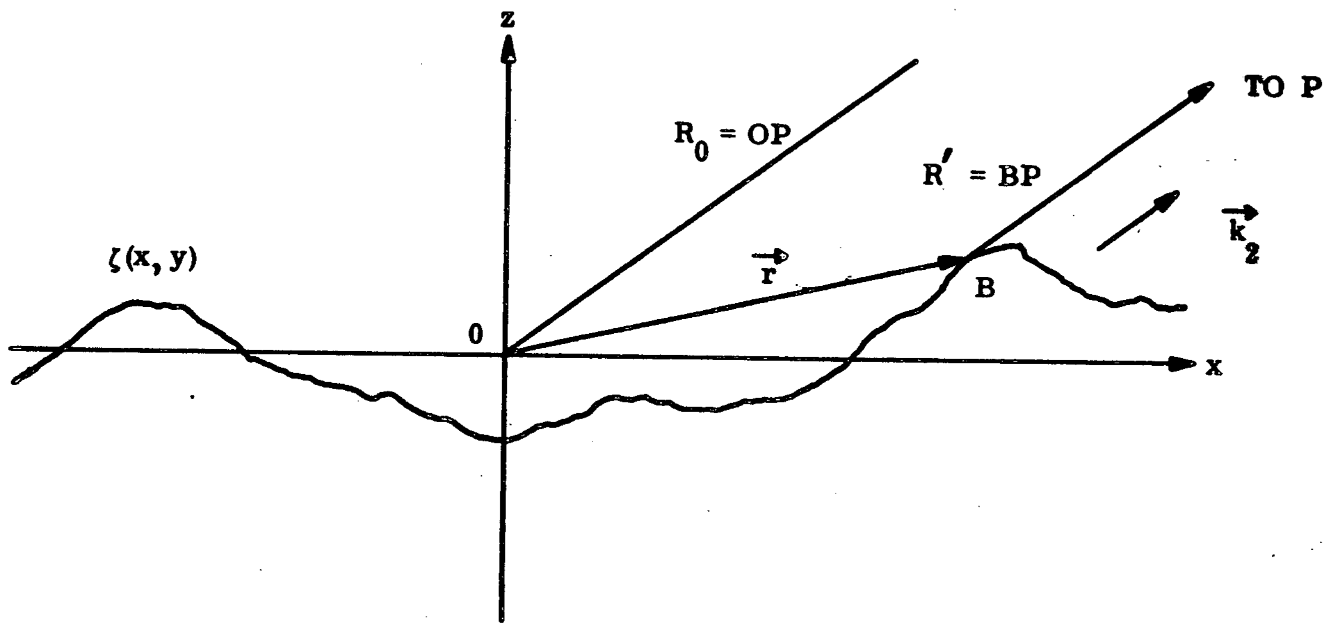

Figure 2-1. Geometry for Scattering From a Rough Surface

2.1 -- Continued.

The angles θ_1, θ_2, and θ_3 are indicated in Figure 2-2. $\vec{k}_2$ is in the direction of reflection at the surface, and $k_2 = |\vec{k}_2| = k_1$.

Let P be the point of observation, and let R′ be the distance from P to a point x, y, ζ (x, y) on the surface S. Then the amplitude E_2 of the scattered field is given by

$$E_2 (P) = \frac{1}{4\pi} \iint_S \left(E \frac{\partial \psi}{\partial n} - \psi \frac{\partial E}{\partial n} \right) ds \tag{3}$$

where

$$\psi = \frac{e^{ik_2 R'}}{R'}$$

and E and $\partial E / \partial n$ are the field and its normal derivative on S. If P is at a great distance from S, then we can make the far field approximation

$$k_2 R' \approx k_2 R_0 - \vec{k}_2 \cdot \vec{r}$$

where R_0 is the distance from the origin to P (see Figure 2-1). The field on the surface and its normal derivative are found by means of the Kirchhoff approximation (B(11), B(12), section 3.1). [B(11) refers to Equation 11 of Reference 1.]

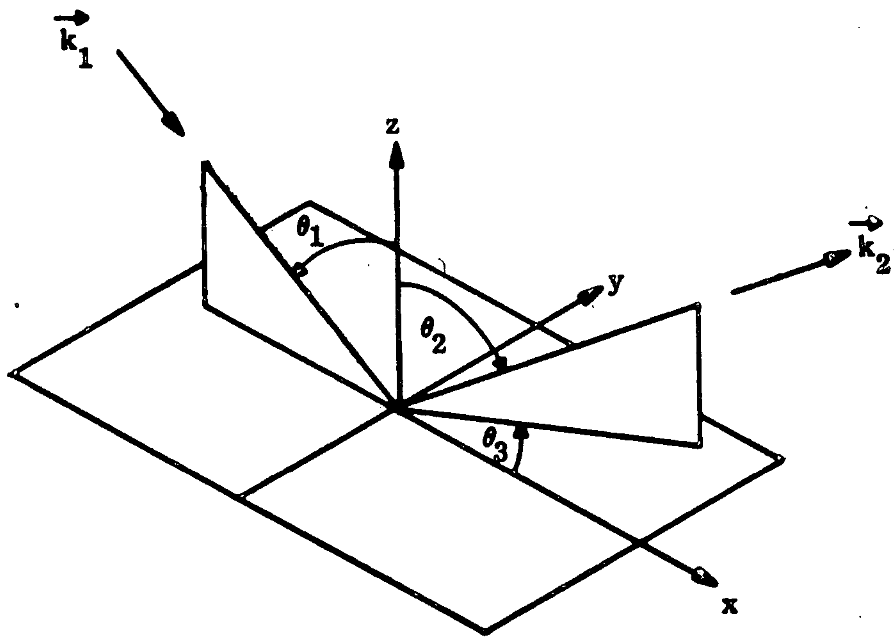

Figure 2-2. Definition of Scattering Angles

2.1 -- Continued.

$$E = (1 + R) E_1$$

and

$$\frac{\partial E}{\partial n} = i(1 - R) E_1 \vec{k}_1 \bullet \vec{n}$$

where $\vec{n}$ is the normal to the surface $z = \zeta(x, y)$, and R is the reflection coefficient of a smooth plane which depends on the incident angle, the reflecting material, and the polarization of $\vec{E}_1$. The reflection coefficients were originally derived by Fresnel and are often referred to as the Fresnel coefficients. See Section 2.3 for a discussion of the Fresnel coefficients.

Applying these approximations to Equation (3) we obtain

$$E_2(P) = \frac{i\, e^{ikR_0}}{4\pi R_0} \iint_S (R\vec{v} - \vec{p}) \bullet \vec{n} \quad e^{i\vec{v} \cdot \vec{r}} \, ds. \tag{4}$$

where

$$\vec{v} = \vec{k}_1 - \vec{k}_2$$

$$\vec{p} = \vec{k}_1 + \vec{k}_2$$

2.1 -- Continued.

It is convenient to normalize E_2 by E_{20}, the amplitude of the field reflected by a smooth, perfectly conducting plane. Setting $R = 1$ and $\theta_1 = \theta_2$ in Equation (4) yields

$$E_{20} = \frac{ikA\cos\theta_1}{\pi R_0} e^{ikR_0}$$

where A is the area of the projection of S onto the x, y plane. We obtain

$$\rho \equiv \frac{E_2}{E_{20}} = \frac{1}{A\cos\theta} \int_{-Y}^{Y}\int_{-X}^{X} \left(a\frac{\partial\zeta}{\partial x} + b\frac{\partial\zeta}{\partial y} - c\right) e^{i\vec{v}\cdot\vec{r}}\, dx\, dy$$

(cf. B(1), Section 3.2), where

$$\vec{v} = k\left[\sin\theta_1 - \sin\theta_2\cos\theta_3,\ -\sin\theta_2\sin\theta_3,\ -\cos\theta_1 - \cos\theta_2\right]$$

$$a = (1-R)\sin\theta_1 + (1+R)\sin\theta_2\cos\theta_3$$

$$b = (1+R)\cos\theta_2 - (1-R)\cos\theta_1$$

$$c = (1+R)\sin\theta_2\sin\theta_3$$

$$ds = \sqrt{1+\zeta_x^{\ 2}+\zeta_y^{\ 2}}\; dx\, dy$$

and the region of integration has been approximated by a rectangle in the x, y plane.

 B

2.1 -- Continued.

In order to make progress with this formulation, it is necessary to treat a, b, and c as constants, in which case, (cf. B(10) Section 3.2 and B(7) Section 5.4)

$$\rho = \langle R \rangle \frac{F_3}{A} \int_{-Y}^{Y} \int_{-X}^{X} e^{i\vec{v} \cdot \vec{r}} \, dx\, dy \tag{5}$$

where

$$F_3 = \frac{1 + \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos\theta_3}{\cos\theta_1 \left(\cos\theta_1 + \cos\theta_2\right)}$$

$\langle R \rangle$ = average of R over the surface

$$\langle R \rangle = \int_{-\infty}^{\infty} P_1(\zeta') \, R(\zeta') \, d\zeta'$$

we approximate $\langle R \rangle \approx R\left(\theta = \theta_1\right)$.

The average value of ρ is found to be

$$\langle \rho \rangle = \frac{\langle R \rangle F_3}{A} \int_{-Y}^{Y} \int_{-X}^{X} e^{ixv_x + iyv_y} \langle e^{i\zeta v_z} \rangle \, dx\, dy$$

$$= \rho_0 \, \chi\left(v_z\right) \tag{6}$$

2.1 -- Continued.

where

$$\rho_0 = \frac{\langle R \rangle F_3}{A} \frac{\sin v_x X}{v_x} \frac{\sin v_y Y}{v_y}$$

$$\chi(v_z) \equiv \langle e^{i\zeta v_z} \rangle = \int_{-\infty}^{\infty} e^{i v_z \zeta} P_1(\zeta)\, d\zeta$$

is the characteristic function of the probability density P_1.

For the normal distribution we have (B(3), Section 5.3)

$$\chi(v_z) = e^{-1/2\, \sigma^2 v_z^2} \tag{7}$$

The average power received is proportioned to $\rho\rho^*$; therefore we require an approximation of $\rho\rho^*$:

$$\langle \rho\rho^* \rangle = \frac{\langle |R|^2 \rangle F_3^{\,2}}{A^2} \int_{-Y}^{Y} \int_{-Y}^{Y} \int_{-X}^{X} \int_{-X}^{X} e^{\left[i v_x (x_1 - x_2) + i v_y (y_1 - y_2)\right]}$$

$$\cdot\ \chi_2 \left(v_z, - v_z\right) dx_1\, dx_2\, dy_1\, dy_2 \tag{8}$$

2.1 --Continued.

where

$$\chi_2(u, v) = \langle e^{iv\zeta_1} \; e^{iu\zeta_2} \rangle$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{iv\zeta_1} \; e^{iu\zeta_2} \; P_2(\zeta_1, \zeta_2) \, d\zeta_1 \, d\zeta_2$$

For the probability density given by Equation (2), we have (B(40), Appendix C)

$$\chi_2(u, v) = e^{\left[-1/2\,\sigma^2\left(u^2 + 2Cuv + v^2\right)\right]}$$

so that

$$\chi_2\left(v_z, -v_z\right) = e^{\left[-\sigma^2\,(1-C)\,v_z^{\,2}\right]} \tag{9}$$

It is convenient because of convergence problems of certain integrals below to calculate

$$D(\rho) \equiv \langle \rho\rho^* \rangle - \langle \rho \rangle \langle \rho^* \rangle$$

2.1 -- Continued.

$$D(\rho) = \frac{\langle |R|^2 \rangle F_3^{\,2}}{A^2} \int_{-Y}^{Y} \int_{-Y}^{Y} \int_{-X}^{X} \int_{-X}^{X} e^{\left[i v_x (x_1 - x_2) + i v_y (y_1 - y_2)\right]}$$

$$\left[\chi_2 (v_z, -v_z) - \chi(v_z) \chi^*(v_z) \right] dx_1\, dx_2\, dy_1\, dy_2 \tag{10}$$

In order to simplify this integral, Beckman introduces two transformations. The first is defined by

$$\tau_1 = x_1 - x_2$$

$$u_1 = x_1 + x_2$$

$$\tau_2 = y_1 - y_2$$

$$u_2 = y_1 + y_2$$

whereupon Equation (10) becomes

$$D(\rho) = \frac{\langle |R|^2 \rangle F_3^{\,2}}{A^2}\, 4XY \int_{-2Y}^{2Y} \int_{-2X}^{2X} e^{\,i v_x \tau_1 + i v_y \tau_2}$$

$$\left[\chi_2 (v_z, -v_z) - \chi (v_z)\, \chi^* (v_z) \right] d\tau_1\, d\tau_2$$

2.1 -- Continued.

(Certain terms have been neglected here, Beckman justifies this by claiming that the quantity in brackets is very small except for τ_1 and τ_2 near zero.) The second transformation is:

$$\tau_1 = \tau \cos \varphi$$

$$\tau_2 = \tau \sin \varphi$$

which yields

$$D(\rho) = \frac{\langle |R|^2 \rangle F_3^2}{A} \int_0^{\infty} \int_0^{2\pi} e^{i v_x \tau \cos \varphi + i v_y \tau \sin \varphi}$$

$$\cdot \left[\chi_2 (v_z, v_z) - \chi (v_z) \chi (v_z) \right] \tau \, d\varphi \, d\tau$$

where now the region extends over the entire xy plane, the justification being that the quantity in brackets is very small for large τ. The formula

$$J_0 \left(\sqrt{x^2 + y^2} \right) = \frac{1}{2\pi} \int_0^{2\pi} e^{ix \cos \varphi + iy \sin \varphi} \, d\varphi$$

then gives

$$D(\rho) = \frac{2\pi F_3^2 \langle |R|^2 \rangle}{A} \int_0^{\infty} J_0 \left(\tau \, v_{xy} \right) \left[\chi_2 (v_z, -v_z) - \chi \chi^* \right] \tau \, d\tau \qquad (11)$$

2.1 -- Continued.

where $$v_{xy} = \sqrt{v_x^2 + v_y^2}$$

(There is a misprint in B(33), Section 5.2; A^2 should be A. Also $\langle |R|^2 \rangle$ appears in our formula because of B(8), Section 5.4.) The function in brackets is, from Equation (7) and Equation (9)

$$\chi_2 (v_z, -v_z) - \chi(v_z)\, \chi^* (v_z) = e^{-g\left(1 - e^{-\tau^2/T^2}\right)} - e^{-g}$$

and

$$g = v_z^2 \sigma^2 = \left[\frac{2\pi\sigma}{\lambda} \left(\cos\theta_1 + \cos\theta_2\right)\right]^2$$

At this point previous authors (Beckman, Section 5.3) have taken an approximation to Equation (11) which is valid only when g is very large. Since all values of g may be important in a given problem, we choose to evaluate Equation (11) numerically.

2.2 Scattering From the Surface of the Earth.

The results of the previous section will now be applied to obtain approximations of the field at the receiver after the reflection of a spherical wave from the surface of the earth. We imagine the total surface of reflection to be broken up into surface elements δS, each of which is approximated by a section of a plane. Each of these plane surfaces reflects a certain amount of energy to the receiver according to the results of the previous section. These amounts are added to yield the total energy reflected into the receiver.

2.2 -- Continued.

The reflecting surface is indicated in Figure 2-3.

Recall that

$$D(\rho) = \langle \rho\rho^* \rangle - \langle \rho \rangle \langle \rho^* \rangle$$

and

$$\rho = \frac{E_2}{E_{20}}$$

where

$$E_{20} = \frac{\delta A \, e^{ikR_0} \cos \theta_1}{\pi R_0}$$

and δA is the area of the surface element δS.

Defining

$$D(E_2) = \langle E_2 \, E_2^* \rangle - \langle E_2 \rangle \langle E_2^* \rangle$$

we have

$$D(E_2) = |E_{20}|^2 \; D(\rho)$$

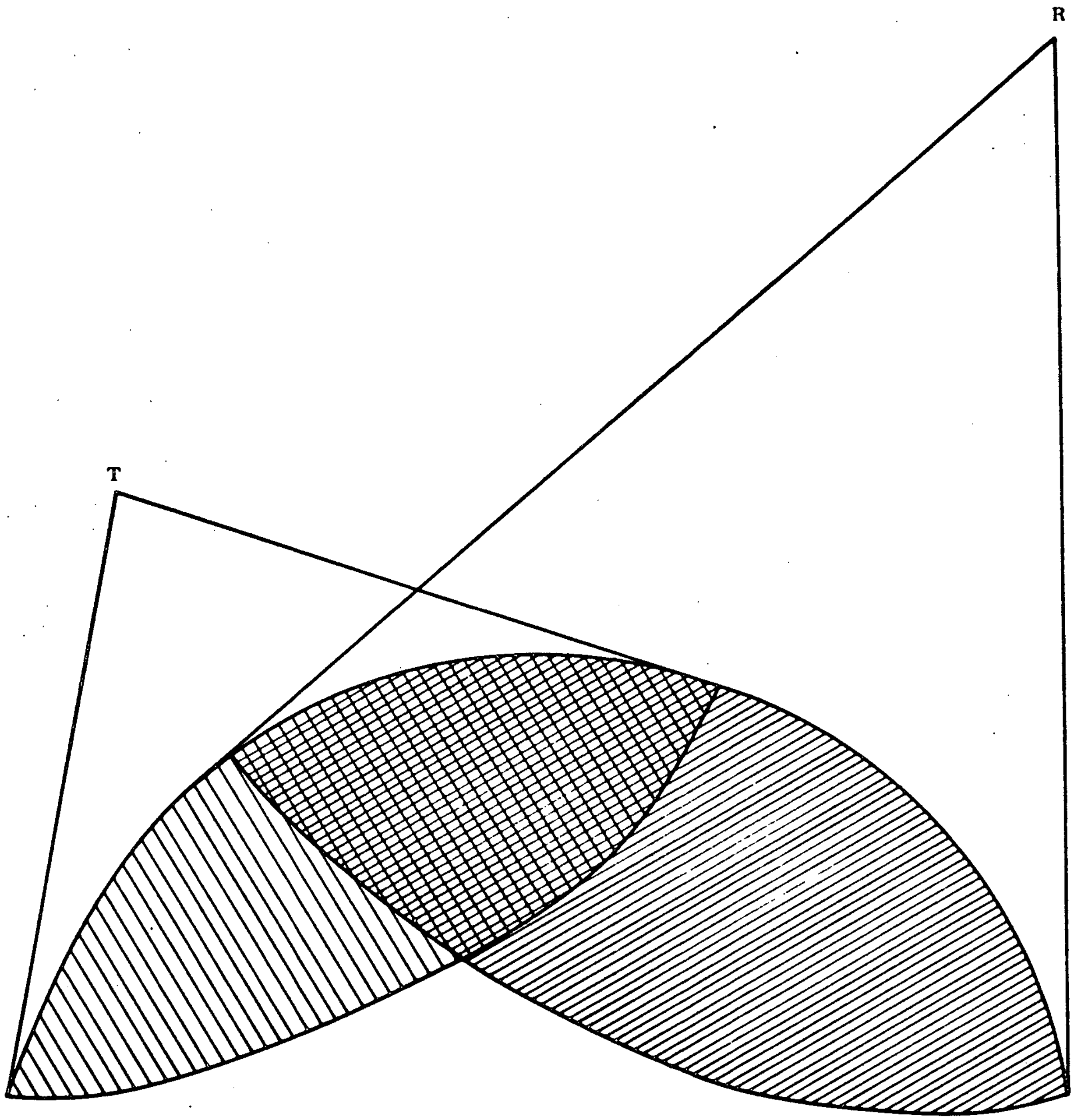

Figure 2-3. Surface on the Earth "Seen" by Both the Transmitter and the Receiver

2.2 -- Continued.

which from Equation (11) is

$$D(E_2) = \delta A\, S(\theta_1)\, D^2(r_1, r_2, \theta_1)\, \langle |R| \rangle^2 \left(\frac{\cos\theta_1}{\pi R_0}\right)^2$$

$$\bullet\ 2\pi F_3^{\,2} \int_0^\infty J_0(v_{xy}\tau)\left[X_2 - XX^*\right] \tau\, d\tau \qquad (12)$$

where we have multiplied by the divergence coefficient $D(r_1, r_2, \theta_1)$ (cf. B(8), Section 11.3) to take into account the curvature at the element δS (r_1 and r_2 are the distances from the transmitter and receiver) and by $S(\theta_1)$, the shadowing function, which will be discussed in Section 2.4. The shadowing function accounts for the shadow cast by one part of the surface onto another part of the surface.

The total power at the receiver is then $\langle E_2 E_2^* \rangle = \sum_i D_i(E_2) + \langle E_2 \rangle \langle E_2 \rangle^*$

which, from Equation (12), is approximately

$$\langle E_2 E_2^* \rangle = \frac{2}{\pi} \iint_S D^2(r_1, r_2, \theta_1)\, S(\theta_1) \left(\frac{\cos\theta_1 F_3}{R_0}\right)^2$$

$$\langle |R|^2 \rangle \int_0^\infty J_0(v_{xy}\tau)\, \left[X_2 - XX^*\right] \tau\, d\tau\, dS$$

$$+ \langle E_2 \rangle \langle E_2 \rangle^* \qquad (13)$$

2.2 -- Continued.

This three-fold integral must be evaluated numerically. For this purpose it is convenient to introduce the coordinate system indicated in Figure 2-4. B is the specular point (the point at which $\theta_1 = \theta_2$, $\theta_3 = 0$), and P is an arbitrary point in the scattering region, A and C are the earth subpoints for the transmitter T and the receiver R. We measure P by the angles θ and φ which are obtained as follows. Construct the great circle through P which is perpendicular to the great circle ABC at M. Define $\theta = BM$ and $\varphi = MP$. The region of integration is the intersection of the two spherical caps defined by those points P for which $\beta < \beta_o$, where $\cos \beta_o = RE/RE + H$ and $\gamma < \gamma_o$, where $\cos \gamma_o = RE/RE + H1$. From the identities of spherical trigonometry, it is easily shown that

$$dS = \cos \varphi \, d\theta \, d\varphi$$

In order to find $\langle E_2 E_2^* \rangle$ (see Equation (13)) we add $\langle E_2 \rangle \langle E_2^* \rangle$ to $D(E_2)$. Since $\langle E_2 \rangle$ is nearly zero except for specular reflection, we may determine $\langle E_2 \rangle$ by modifying E_1 by the path loss together with an appropriate reflection coefficient.

This reflection coefficient is given by (cf. B(4) Section 12.3).

$$R_s = \langle \rho_s \rangle DR$$

where R is the reflection coefficient of a smooth plane earth, (see Section 1.3), D is the divergency coefficient which takes account of the earth's curvature, and ρ_s is the value of $\langle \rho \rangle$ at the specular point, which is given by $\exp^{-1/2\,g}$, from Equation (6).

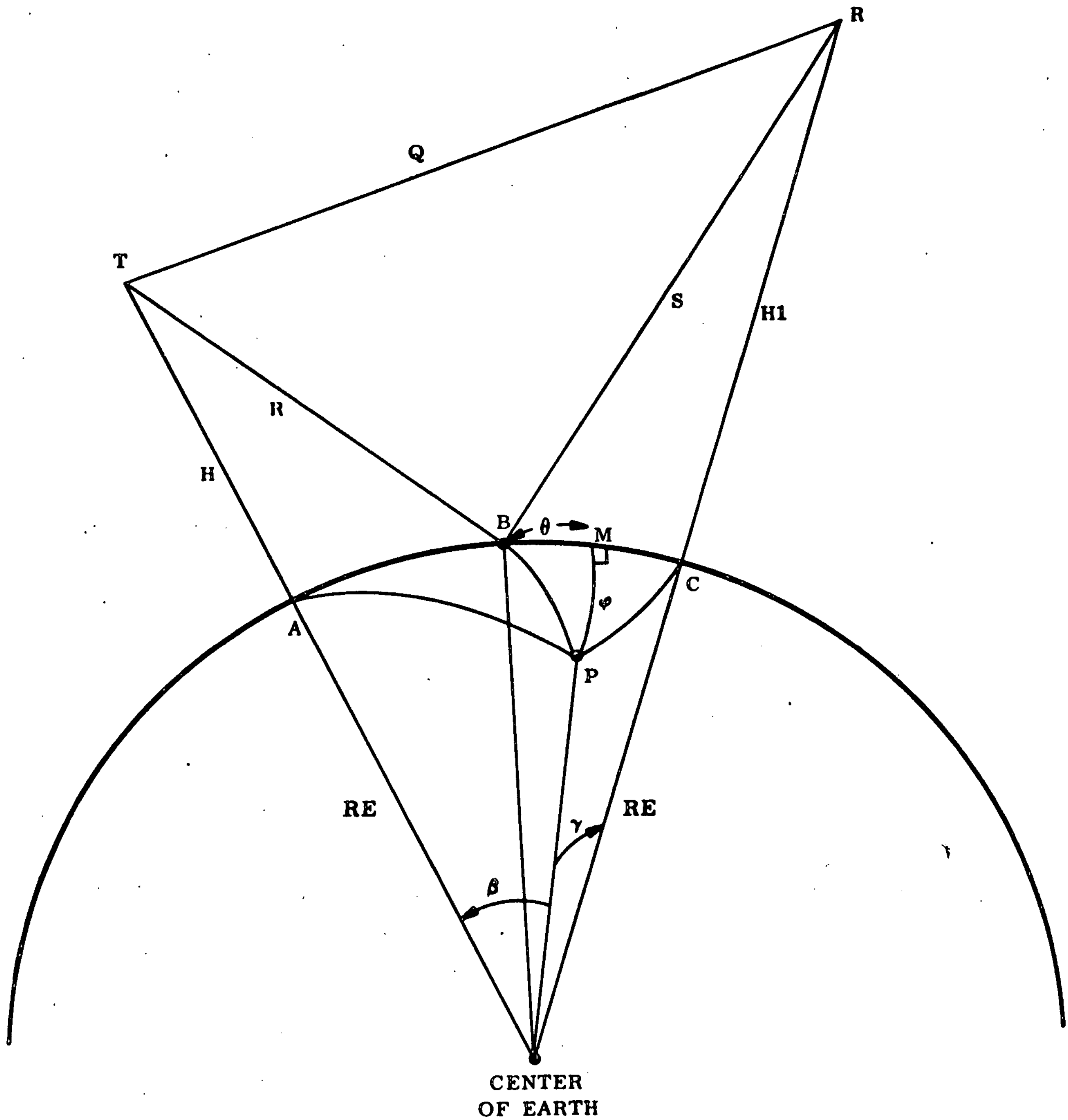

Figure 2-4. Coordinates System and Associated Angles and Distances

2.2 -- Continued.

We have included also a shadowing function $S(\theta_1)$ (cf. Section 2.4) so that

$$\langle E_2\rangle\langle E_2^*\rangle = \frac{e^{-g}}{(R+S)^2}\; D^2 R_o^2\, S(\theta_1)^2$$

where R and S are the distances of the transmitter and receiver from the specular point (see Figure 2-4).

2.3 Fresnel Reflection Coefficients.

The Fresnel coefficients which are used in the calculation are the exact expressions.[2] The reflection coefficient for the component of the wave with electric vector perpendicular to the plane of incidence is given by

$$(R_\perp)^2 = \frac{(\mu_2 q - \mu_1 \alpha_2 \cos\theta_1)^2 + \mu_2^2 p^2}{(\mu_2 q + \mu_1 \alpha_2 \cos\theta_1) + \mu_2^2 p^2}$$

where θ_1 is the incident angle from the normal, and the subscript 2 refers to air and 1 refers to the reflecting surface material. p and q are found from the relations

$$q^2 - p^2 = \alpha_1^2 - \beta_1^2 - \alpha_2^2 \sin^2\theta_1$$

$$q^2 + p^2 = \left[4\alpha_1^2 \beta_1^2 + \left(\alpha_1^2 - \beta_1^2 - \alpha_2^2 \sin^2\theta_1\right)^2\right]^{1/2}$$

2.3 -- Continued.

and α and β are given by

$$\alpha = \omega \left[\frac{\mu\epsilon}{2} \left(\sqrt{1 + \frac{\sigma^2}{\epsilon^2 \omega^2}} + 1 \right) \right]^{1/2}$$

$$\beta = \omega \left[\frac{\mu\epsilon}{2} \left(\sqrt{1 + \frac{\sigma^2}{\epsilon^2 \omega^2}} - 1 \right) \right]^{1/2}$$

with the 1 or 2 subscript specifying which value of μ, ϵ and σ to use. μ, ϵ and σ are the permittivity, permeability and conductivity in rationalized MKS units. The phase change $\delta_\perp$ is given by

$$\tan \delta_\perp = \frac{2\mu_1 \mu_2 \alpha_2 \, p \cos \theta_1}{\mu_1^2 \alpha_2^2 \cos^2 \theta_1 - \mu_2^2 \left(q^2 + p^2\right)}$$

For the E field component parallel to the plane of incidence

$$R_{\parallel}^2 = \frac{\left[\mu_2\left(\alpha_1^2 - \beta_1^2\right)\cos \theta_1 - \mu_1 \alpha_2 q\right]^2 + \left[2\mu_2 \alpha_1 \beta_1 \cos \theta_1 - \mu_1 \alpha_2 p\right]^2}{\left[\mu_2\left(\alpha_1^2 - \beta_1^2\right)\cos \theta_1 + \mu_1 \alpha_2 q\right]^2 + \left[2\mu_2 \alpha_1 \beta_1 \cos \theta_1 + \mu_1 \alpha_2 p\right]^2}$$

and the phase change $\delta_{\parallel}$ is given by

$$\tan \delta_{\parallel} = \frac{2\mu_1 \mu_2 \alpha_2 \, p \left(q^2 + p^2 - \alpha_2^2 \sin^2 \theta_1\right) \cos \theta_1}{-\left[\mu_1^2 \alpha_2^2 \left(q^2 + p^2\right) - \mu_2^2 \left(\alpha_1^2 + \beta_1^2\right)^2 \cos^2 \theta_1\right]}$$

2.3 -- Continued.

In the case when $\sigma_1/\epsilon_1\omega << 1$ the reflection coefficients reduce to

$$R_\perp \approx \frac{\left(K_{e1} - K_{e2}\sin^2\theta_1\right)^{1/2} - \sqrt{K_{e2}}\cos\theta_1}{\left(K_{e1} - K_{e2}\sin^2\theta_1\right)^{1/2} + \sqrt{K_{e2}}\cos\theta_1}$$

$$R_\parallel \approx \frac{K_{e1}\cos\theta_1 - \sqrt{K_{e2}}\left(K_{e1} - K_{e2}\sin^2\theta_1\right)^{1/2}}{K_{e1}\cos\theta_1 + \sqrt{K_{e2}}\left(K_{e1} - K_{e2}\sin^2\theta_1\right)^{1/2}}$$

where

$$K_e = \frac{\epsilon}{\epsilon_o}$$

and ϵ_o is the permeability of free space. This approximation is used in Beckman and Spizzachino[1] with K_e replaced by

$$K_e = \frac{\epsilon}{\epsilon_o} + i\,60\,\lambda\,\sigma$$

For the approximation with $K_e = \epsilon/\epsilon_o$, Stratton[2] shows that reasonable values are obtained for $R_\perp$ and $R_\parallel$ for frequencies above 1 MHz and below 10^4 Mhz. The exact expressions for $R_\perp$ and $R_\parallel$ are valid at any frequency.

2.4 Shadowing Function.

The shadowing function, $S(\theta_1)$, is included in the calculation of $\langle EE^* \rangle$ and $\langle E \rangle \langle E \rangle^*$ to account for the screening of parts of the surface by other parts as shown in Figure 2-5. The shaded area in Figure 2-5 does not contribute to the scattering area and thus any function which contains a factor of area, A, should be modified to contain $S(\theta_1)$ A. $S(\theta_1)$ is defined as

$$S \theta_1 = \langle S (x\ y) \rangle$$

where S (x y) is one at a point which is illuminated and zero at a point which is not illuminated.

We use a shadowing function derived by Beckman.[3] For a normally distributed surface $S(\theta_1)$ is given by

$$S(\theta_1) = e^{\left[-\frac{1}{4} \tan \theta_1 \operatorname{erfc}\left(k \cot \theta_1\right)\right]}$$

where

$$k = \frac{T}{2\sigma}$$

T = correlation length

σ = RMS value of the roughness

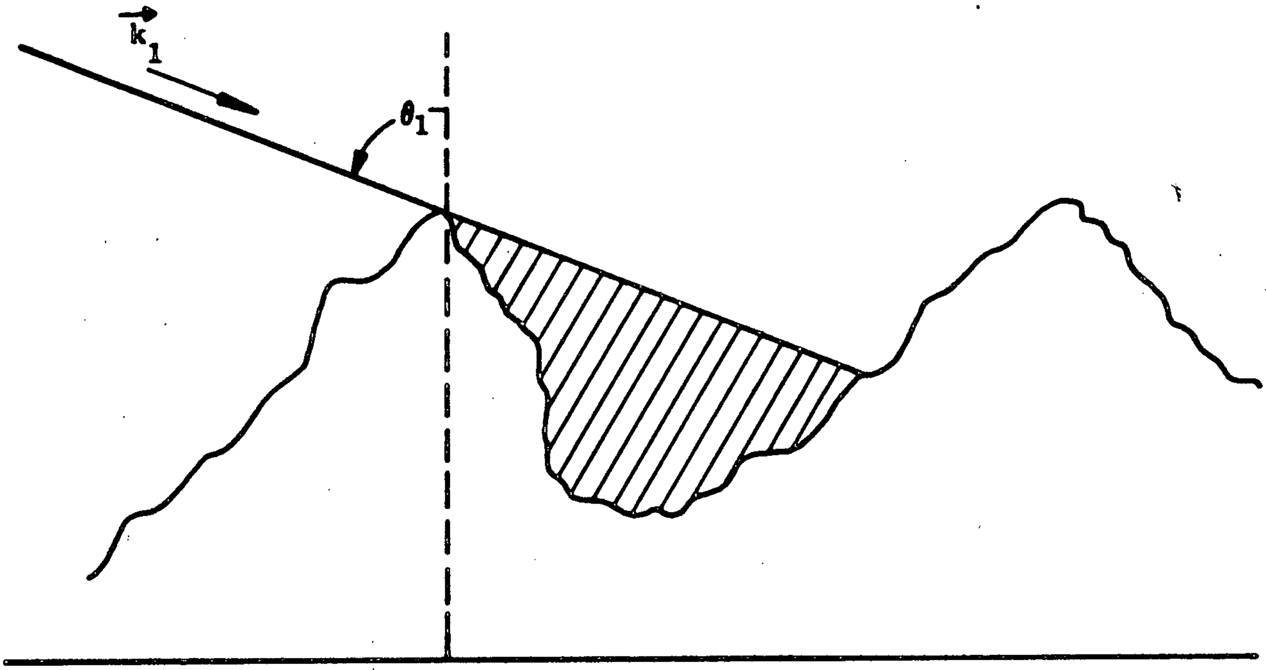

Figure 2-5. Shadowing of a Random Rough Surface

2.5 Discussion of Approximations.

Several approximations are present in the formulation we have presented. Here we discuss these and indicate areas where refinements might be made in the future.

a. The Kirchhoff approximation was applied to the integral representation (Equation (3)) of the field $E_2(P)$ to obtain the field E on the surface. This requires that the radius of curvature of the surface (which is approximately σ/T) is small compared to a wavelength; hence, the approximation breaks down if the surface includes sharp edges. Brekhovskilch (B(2) Section 3.3) gives as a criterion $4\pi r \cos\theta >> \lambda$, where r is the radius of curvature, and θ the local angle of incidence.

b. The point of observation must be far from the scattering surface. This approximation was invoked to obtain k_2R' in the expression for ψ in Equation (3). For satellite communications this approximation will always be satisfactory.

c. The Fresnel coefficient and the function $F_3\left(\theta_1, \theta_2, \theta_3\right)$ appearing in Equation (4) have been replaced by their average value and removed from under the integral sign in Equation (5).

d. Two approximations have been introduced in the transformation of the integral Equation (10). In the integration by parts of Equation (10) the boundary terms have been neglected. Next, the transformation $\tau_1 = \tau \cos\varphi$ and $\tau_2 = \tau \sin\varphi$ gives rise to two terms, one of which is Equation (11), and the other has been neglected.

2.5 -- Continued.

If the correlation distance T is small compared to the diameter of the surface element δS, then the factor $\chi_2(v_z, -v_z) - \chi(v_z)\chi^*(v_z)$ will be small except near $\tau = 0$. This fact makes the neglected term small.

e. The scattering formulation presented here considers only the amplitudes of the incident and reflected fields; thus, no polarization information about the scattered field is available. The only polarization information comes from the Fresnel reflection coefficients. A vector theory for scattering of a plane wave from a plane surface with random irregularities has been discussed in numerous articles.[4, 5]

f. Shadowing has been taken into account as suggested by Beckman;[3] this has been discussed in Section 2.3.

g. Multiple scattering has not been considered.

h. While the basic theory of Section 2.1 assumes a plane wave incident on a plane reflector, we have modeled a point source incident on a sphere by breaking up the surface of the sphere into elements which are approximated by plane sections. The field incident on each section may then be approximated by a plane wave. The resultant reflections are added up to yield the total field at the receiver. Some authors[1, 6] have integrated over a plane earth and modified the result by the divergence coefficient; we call this the flat earth approximation. This procedures does not satisfy our intuition, and there are other difficulties associated with it. First,

2.5 -- Continued.

the region of integration is completely unspecified. Second, the far field approximation is violated. We have attempted to calculate the received field using this approximation; the differences in the results are dramatic in most cases, as indicated by Figure 3-1.

i. The three-fold integral Equation (11) has been computed numerically. In Beckman[1] approximations are obtained for small and large values of g, where g is the roughness criterion. These approximations are not valid for moderate values of g, and since all values of g are liable to arise, we have not used these approximations.

j. The surface height function is assumed to be normally distributed Equation (1).

2.6 Pulse Delay and Broadening.

To evaluate the effect of multipath on a communication system, it is necessary to know the time delay between the multipath and the direct path and to know to what extent a pulse is broadened in time by reflection from the earth's surface. The time delay between the direct and multipath pulses is easily calculated from the geometry of a given problem. In Figure 2-4 a typical geometry is shown. The time delay is given by

$$T = \frac{R + S - Q}{C}$$

where C is the velocity of light.

2.6 -- Continued.

To calculate pulse broadening it is necessary to divide the scattering surface into annular regions, each of which has a nearly constant time delay. To calculate the received field, the surface integral in Equation (13) is evaluated over this annular region.

3. RESULTS OF CALCULATIONS.

In this section some results of calculations using the theory of Section 2 are discussed. The only effects included in these calculations are rough surface scattering and space loss. Pulse distortion, Faraday rotation, etc., are not included in the present report results. All are for VHF frequencies and for a terrain corresponding to marshy land.

3.1 Discussion of Results.

Figure 3-1 shows a comparison between the RCA model[6] and our curved earth and flat earth models. Our calculations are performed for a roughness, σ, of 1.0 meters and a correlation distance, T, of 10 meters. In the RCA model the ration σ/T is assumed small and does not enter explicitely into the calculation. The altitude of one satellite is 275 nautical miles and the other is at synchronous altitude. Our curved earth model shows a dip at around 45 degrees which neither of the other two calculations show. This dip is caused by the fact that the diffuse scattering is decreasing while the specular term is increasing as the incident angle increases. Our curves approach zero at 90 degrees due to the divergence term, D, and the shadowing term, $S(\theta_1)$. The RCA curve approaches zero due to the divergence term and due to the fact that it does not contain the specular term.

Figure 3-2 shows the effect of varying the altitude of one satellite while the other stays at 275 nautical miles. Roughness is 1.0 meter and correlation length is 10 meters for all calculations.

Figure 3-3 shows the effects of varying the roughness and correlation length. One satellite is at 275 nautical miles and the other is a synchronous satellite. For one value of roughness, 1 meter, the minimum in the multipath signal moves to smaller incidental angles as the correlation length is increased from 10 meters to 30 meters.

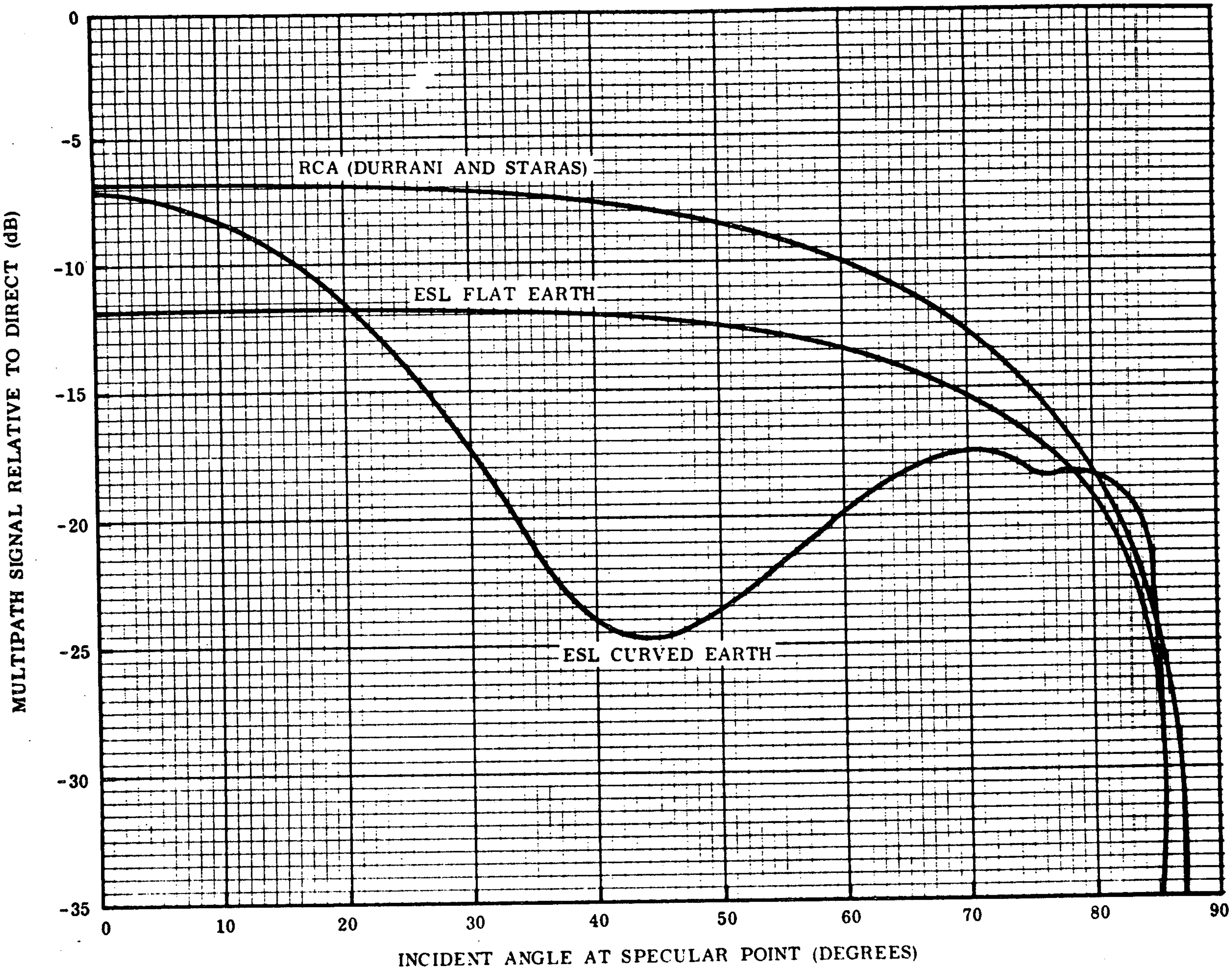

Figure 3-1. Multipath Signals Using Three Different Calculation Schemes

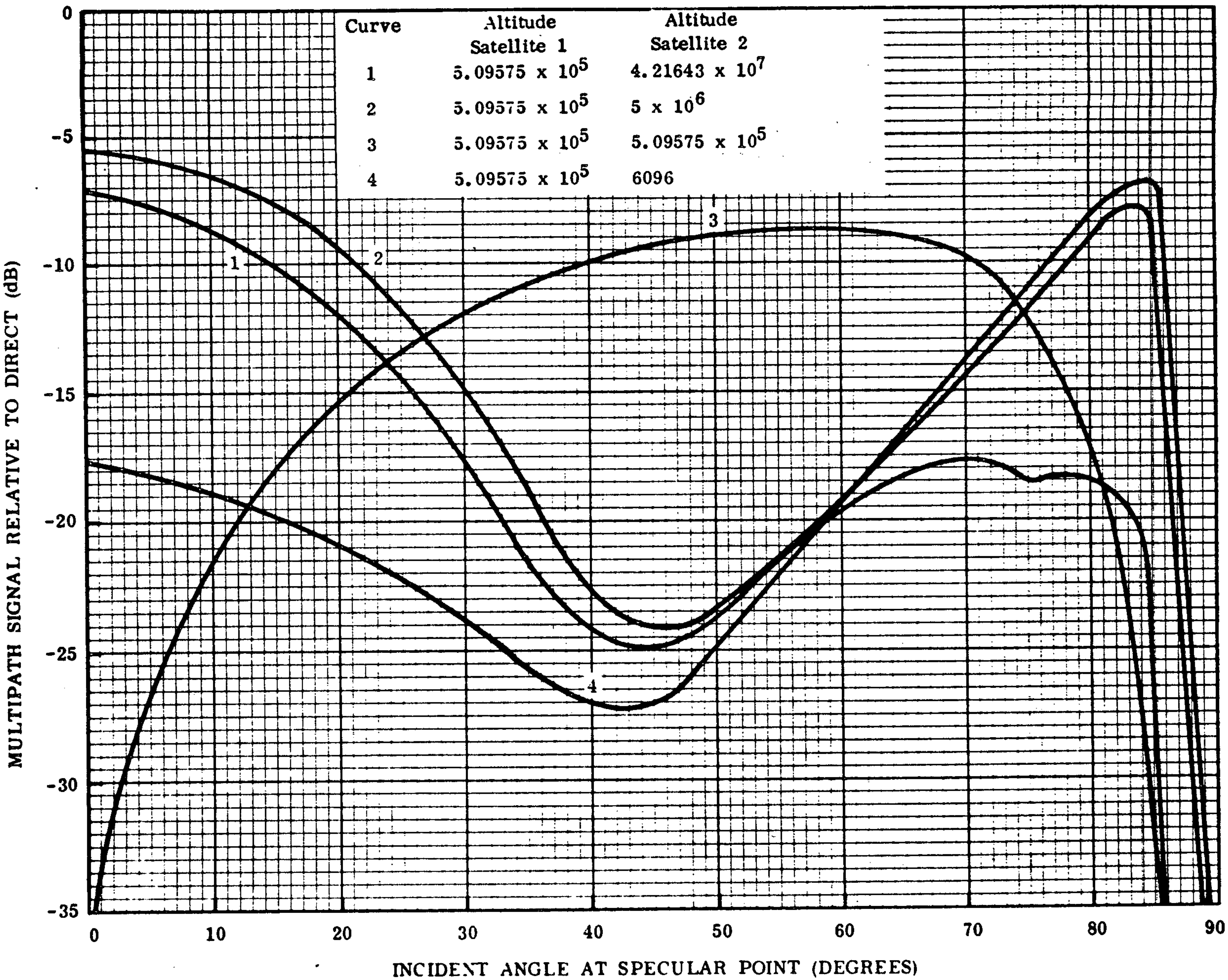

Figure 3-2. Multipath Return for Satellites at Various Altitudes (in Meters)

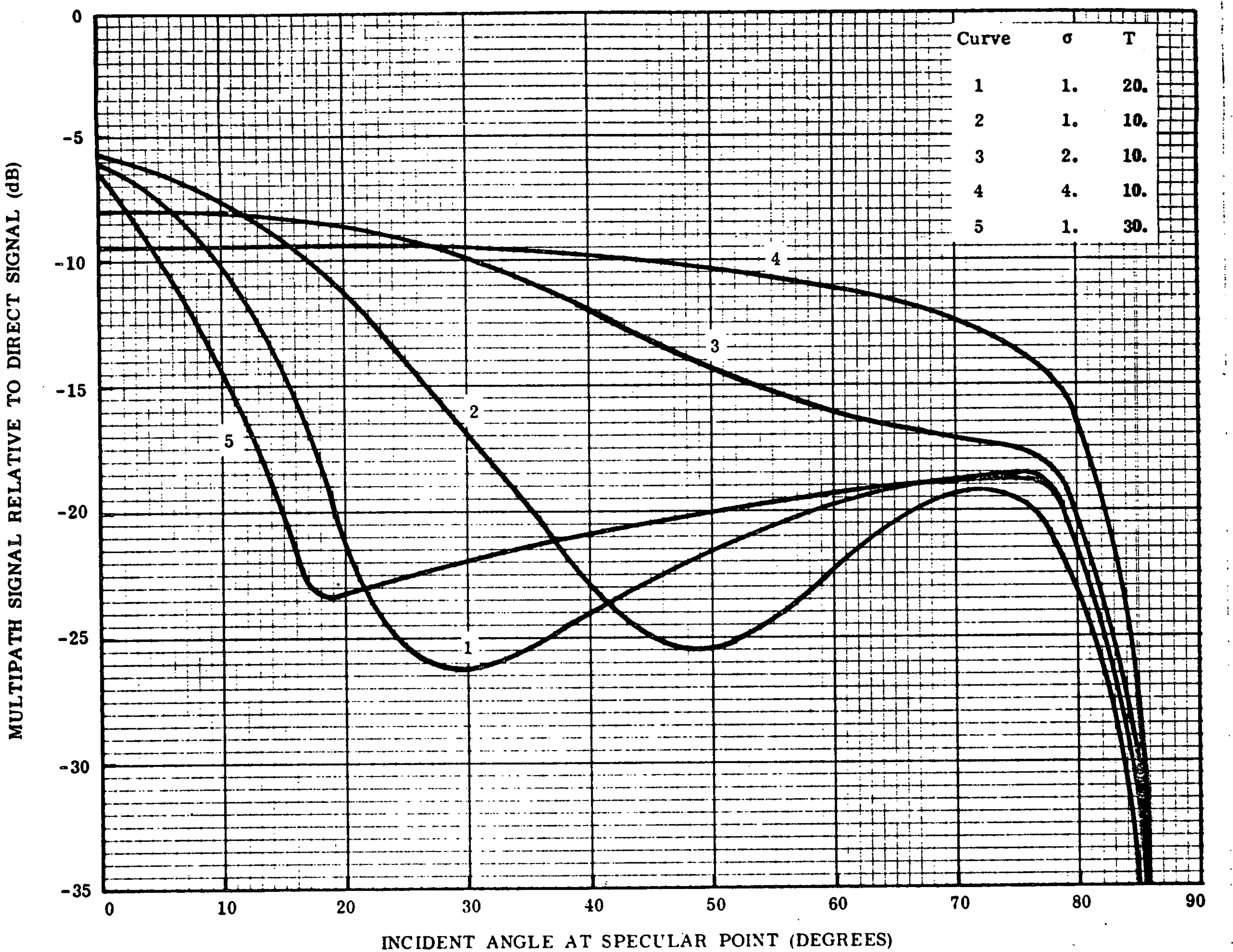

Figure 3-3. Multipath Signal for Various Values of Roughness, σ, and Correlation Length, T

3.1 -- Continued.

It is also seen that for one value of the correlation length, 10 meters, the minimum in the returned signal becomes less pronounced as the roughness goes from 1 meter to 4 meters. It is apparent that there is considerable dependence upon σ and T in the multipath returned signal.

Figure 3-4 shows the returned signal as a function of time for one satellite at 275 nautical miles and the other satellite at synchronous altitude. A pulse duration time of 1 microsecond was used and the returned signal was calculated in 2 microsecond steps out to 12 microseconds. The curves have a maximum in the zero to 2 microsecond region because of the specular signal. Beyond 2 microseconds the signal stays very constant.

3.2 Conclusions.

From Figure 3-1 it is apparent that there are considerable differences in the resulting multipath returns between the flat earth approximation and the more appropriate spherical earth case. This is largely due to the poorly defined scattering area for the flat earth approximation.

The calculation presented in Figure 3-2 shows that there is some dependence of the multipath return on the altitude of the two satellites. This would be expected because of the change in the size of the scattering area and the change in the space loss as the altitudes vary.

Figure 3-3 is quite important because it shows for the first time the exact dependence of the multipath signal on the parameters σ and T. It can be concluded that there is a definite dependence on σ and T and that it should not be ignored in evaluating the effect of multipath on a satellite to satellite communications system.

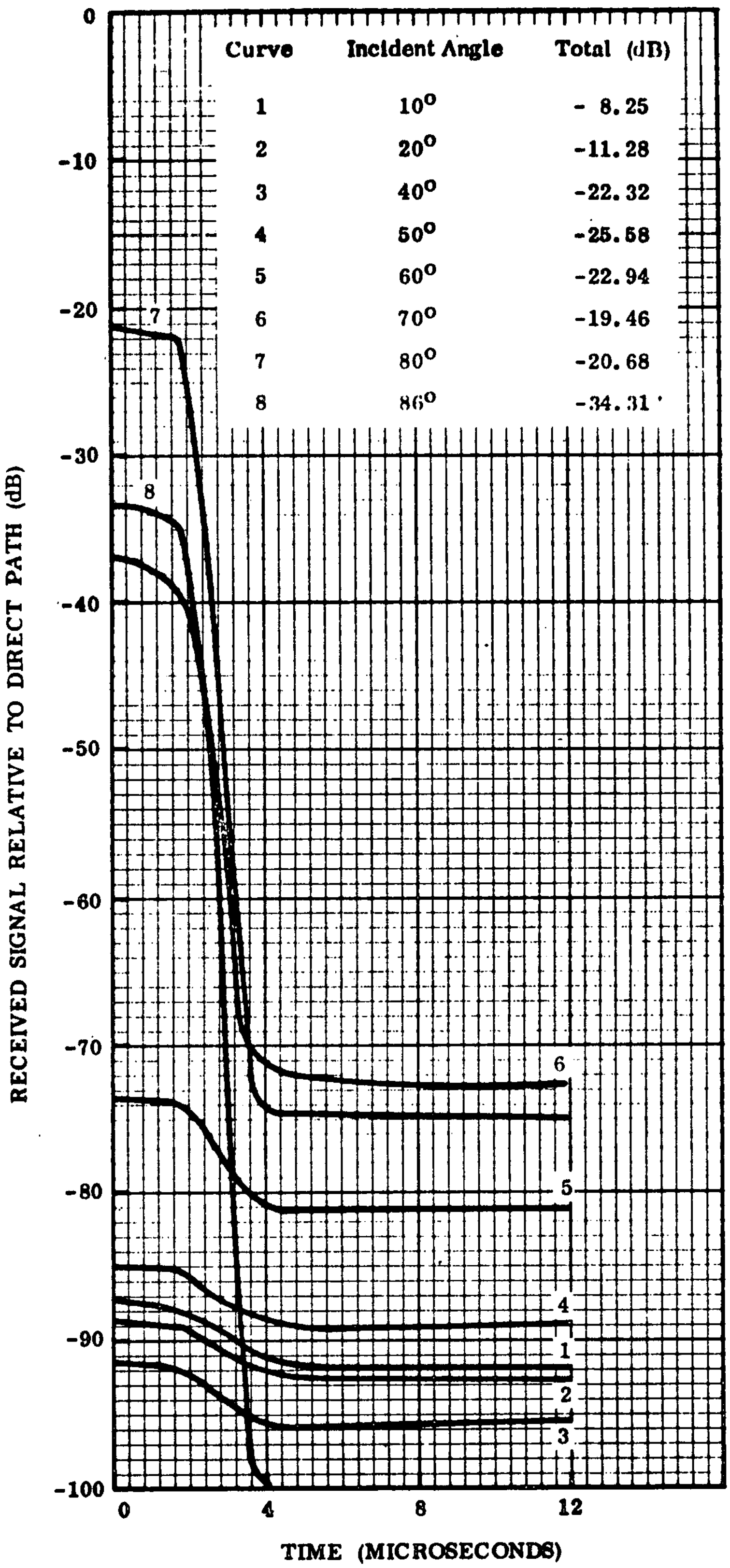

Figure 3-4. Received Signal versus Time for a 1 μs Pulse

3.2 -- Continued.

The pulse versus time calculations show that it is necessary to perform this type of calculation to determine at what satellite position the most multipath return will be received. It can not be concluded that the most serious interference case for a pulsed system occurs when the satellites are overhead. For a CW transmitter the worst case will be dependent upon data rate and pulse duration time, but will occur when the two satellites are directly overhead.

A final conclusion which was reached during the derivation of the theory is that previous calculations based on approximations to Equation (11) are of doubtful value. The reason for this is that the approximations are valid only for the case of very diffuse scattering. As either the roughness gets smaller or the separation between satellites gets greater, these approximations are very suspect.

4. REFERENCES.

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